The Casimir force for passive mirrors

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Abstract

We show that the Casimir force between mirrors with arbitrary frequency dependent reflectivities obeys bounds due to causality and passivity properties. The force is always smaller than the Casimir force between two perfectly reflecting mirrors. For narrow-band mirrors in particular, the force is found to decrease with the mirrors bandwidth.

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1 Introduction

Two reflectors placed in vacuum exert a force onto each other, since the energy stored between them depends on their relative positions. This well known Casimir effect is a macroscopic mechanical consequence of quantum fluctuations of electromagnetic fields. In the standard point of view, the Casimir energy is deduced from the part of vacuum energy which depends on the presence and position of reflecting boundaries [1]. In a local point of view in contrast, the Casimir force is understood as the radiation pressure exerted upon mirrors by vacuum fluctuations [2]. As it is known from theory and experiments in Cavity Quantum Electrodynamics [3], vacuum fluctuations are enhanced or suppressed inside the cavity, depending on whether the field frequency matches a cavity resonance or not. Whereas at resonance the vacuum energy density inside the cavity is increased and thus pushes the mirrors apart, out of resonance vacuum fluctuations are suppressed and the mirrors are attracted to each other. The net Casimir force therefore appears as an average between repulsive and attractive contributions associated respectively with resonant and antiresonant parts of the spectrum.

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In this context, the case of mirrors with frequency dependent reflectivity amplitudes is particularly interesting, not only because it corresponds to the realistic case, but also for conceptual reasons. First, it provides a natural regularisation procedure to dispose of the divergences associated with the infiniteness of vacuum energy [4]. Then, it suggests to choose specific frequency dependences designed to change the balance between attractive and repulsive contributions to the force. For narrow-band mirrors in particular, it may be thought that the Casimir force depends in a sensitive manner on the relative tuning between the mirrors reflectivity bands and cavity resonances and, therefore, on the distance between the mirrors for given reflectivity functions. It might also be hoped that the force reaches values larger than for perfect mirrors or that it can be alternatively attractive or repulsive. If confirmed, these features would allow to design novel and more sensitive experimental demonstrations of the Casimir effect [5].

The aim of the present letter is to discuss these expectations and to analyse them in connection with general properties obeyed by real mirrors, namely causality and passivity. Passivity is a fundamental property related to energy considerations: in the absence of an internal gain mechanism, mirrors are not able to provide energy to the field [6]. Passivity of mirrors then ensures stability of intracavity field. In a first stage we derive properties of the Casimir force evaluated for an arbitrary passive system of frequency dependent mirrors and show that the force never exceeds the value corresponding to the limiting case of perfectly reflecting mirrors. We then concentrate on a cavity formed by two multilayer dielectric mirrors and find that the Casimir force is always attractive and a decreasing function of cavity length. We finally consider narrow-band mirrors designed to disturb the balance between attractive and repulsive contributions to the Casimir force. We derive a simple expression for the force revealing it to be in fact much smaller than the value corresponding to perfect mirrors. In particular the force decreases with the mirrors bandwidth.

For simplicity, we limit ourselves here to calculations in a model two-dimensional space-time. As is well known from the analysis of quantum optical experiments using cavities [7], each transverse cavity mode is correctly described by such a model provided that the size of the mirrors is larger than the spot size associated with the mode and that the mode may be treated in a paraxial approximation. In the particular case of narrow-band mirrors, the calculations presented in this paper correspond therefore to the Casimir force due to a single transverse mode. A more elaborate approach would require a detailed evaluation of the effects of diffraction, accounting in particular for the dependence upon diffraction of the reflection coefficients of each mode [8]. The Casimir force in the realistic four dimensional configuration may be qualitatively evaluated as the product of the two dimensional result obtained in this letter with the number of modes efficiently coupled to the cavity, that is to say the Fresnel number [5].

2 The Casimir force for frequency-dependent mirrors

From now on, we restrict our attention to the simple model of a scalar field in a mono-dimensional space. The Casimir force between two perfectly reflecting mirrors thus scales as the squared inverse of the distance q or of the time of flight τ between the two mirrors [2]

$$F_{\mathrm{P}} = \frac{\pi\hbar}{24c\tau^2} \qquad \tau = \frac{q}{c} \tag{1}$$

More generally, the Casimir force between two partly reflecting and frequency-dependent mirrors may be expressed as follows [4]

$$\begin{aligned} F &= \frac{\hbar}{c}\int_0^\infty \frac{\mathrm{d}\omega}{2\pi}\omega\left(1-g[\omega]\right) \\ g[\omega] &= \frac{1-\left|r_1[\omega]r_2[\omega]\right|^2}{\left|1-r_1[\omega]r_2[\omega]e^{2i\omega\tau}\right|^2} \end{aligned} \tag{2}$$

The upper equations describe in a quantitative way the interpretation presented in the Introduction. They give the Casimir force F as the difference between outer and inner radiation pressure, defined in such a manner that a positive value of the force corresponds to attraction. The density of vacuum energy per mode at frequency ω is given by the expressions $\frac{\hbar\omega}{2c}$ and $\frac{\hbar\omega}{2c}g[\omega]$ respectively outside and inside the cavity. The Airy function $g[\omega]$ depends on the reflection amplitudes r_1 and r_2 of the two mirrors and describes the modification of vacuum energy inside the cavity with respect to the incoming vacuum energy. It corresponds to an enhancement or suppression of vacuum fluctuations inside the cavity depending on whether the field frequency is resonant or non-resonant with a cavity mode [3]. As a result of causality $r_1[\omega]r_2[\omega]$ is analytic for frequencies ω lying in the upper half $\mathrm{Im}\,\omega > 0$ of the complex plane [9].

The expectations which have been discussed in the Introduction may now be stated in a more quantitative manner. Since the Airy function $g[\omega]$ nearly vanishes in the interval between two cavity resonances, the attractive contribution to the Casimir force from this interval ($\Delta\omega \simeq \frac{\pi}{\tau}$ corresponding to the free spectral range) may be approximated to the amount $\frac{\hbar\omega}{2c\tau}$. Inside the cavity resonances in contrast, the Airy function $g[\omega]$ is much larger than unity. At exact resonance, it reaches a maximum value proportional to the cavity finesse. The frequency interval corresponding to the resonance peaks is much smaller than the free spectral range and the ratio between these two quantities is controlled by the cavity finesse. As a consequence, the repulsive contribution corresponding to a resonance peak has nearly the same magnitude as the attractive contribution corresponding to the neighbouring free spectral range. This

implies that attractive and repulsive contributions nearly cancel in the evaluation of the net Casimir force. Actually, for every interval, the attractive and repulsive contributions each largely exceed the net Casimir force F_{P}, at least for high order modes such that $\omega\tau \gg 1$. This is precisely the reason why it might be expected that a Casimir force larger than F_{P} could be obtained by disturbing this fine balance between attractive and repulsive contributions. We will however show that this expectation cannot be met, because of causality and passivity properties of mirrors.

3 The Casimir force between reciprocal passive mirrors

To go further in the discussion of the Casimir force, we have now to specify these properties. In the present section, we will use the minimal assumption that the mirrors are causal and passive systems [6]. We will first concentrate on the properties of a single mirror and then deduce the properties of a pair of mirrors and therefore of the Casimir force.

We consider each mirror as a network with two ports corresponding to the left-hand and right-hand sides of the mirror. We introduce amplitudes $a_{\mathrm{L}}^{\mathrm{in}}$, $a_{\mathrm{L}}^{\mathrm{out}}$, $a_{\mathrm{R}}^{\mathrm{in}}$ and $a_{\mathrm{R}}^{\mathrm{out}}$ which describe the input and output fields at the two ports of the mirror as shown in Fig. 1. These amplitudes are classical numbers representing

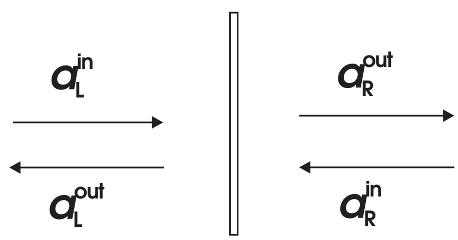

Figure 1: Incoming and outgoing field amplitudes on the left-hand and right-hand side of the mirror.

mean values of the fields evaluated on the physical boundaries of the mirror. The same equation describes also the transformation of the quantum field amplitudes, that is of the annihilation and creation operators [4]. For dissipative mirrors extra terms have to be added which describe Langevin forces associated with loss mechanisms [10]. We will however not discuss these terms in more detail since both cases of dissipative or non dissipative mirrors can be dealt with in exactly the same manner [11].

The field amplitudes are gathered in column vectors a^{in} and a^{out} according

to the rule

$$a = \begin{bmatrix} a_{\rm L} \\ a_{\rm R} \end{bmatrix} \tag{3}$$

We relate input and output field amplitudes through a scattering matrix S, the coefficients of which are two reflection amplitudes r and $\overline{r}$ and a transmission amplitude t

$$a^{\rm out} = S a^{\rm in} \qquad S = \begin{bmatrix} r & t \\ t & \overline{r} \end{bmatrix} \tag{4}$$

Micro-reversibility of the electromagnetic processes taking place inside the mirror entails that the network is reciprocal [12], which means that the S-matrix is symmetrical, the two transmission amplitudes being equal. However this does not imply that the mirror is symmetrical in the exchange of its two ports. Should this further property be also obeyed, the two reflection amplitudes r and $\overline{r}$ would be equal too.

We then introduce the magnetic and electric fields at the two ports of the network (same notations as in eq.(3))

$$h = a^{\rm in} - a^{\rm out} \qquad e = a^{\rm in} + a^{\rm out} \tag{5}$$

These fields are related through an impedance matrix Z directly connected to the S-matrix

$$e = Zh \qquad S = \frac{Z-1}{Z+1} \tag{6}$$

As an immediate consequence of this relation, it follows that

$$1 - SS^\dagger \quad = \quad 2 \frac{1}{Z+1} \left(Z + Z^\dagger \right) \frac{1}{Z^\dagger + 1} \tag{7}$$

We will use this relation in the following to derive an upper bound of the Casimir force between passive mirrors.

Mirrors are electromagnetic networks which obey properties similar to those of electrical networks. Passivity of a mirror means that it does not provide energy to the field when irradiated by arbitrary incident amplitudes. This property may be stated in terms of the power π characterizing this exchange of energy

$$\begin{aligned} &\pi(t) = e(t)^\dagger h(t) = a^{\rm in}(t)^\dagger a^{\rm in}(t) - a^{\rm out}(t)^\dagger a^{\rm out}(t) \\ &\int_{-\infty}^{t} \pi(t')dt' \geq 0 \end{aligned} \tag{8}$$

The impedance-matrix Z of the mirror, considered as a function of complex frequency $\omega = ip$, then obeys the following positivity property [6] for an arbitrary column vector h

$$\mathrm{Re}\, p \geq 0 \Rightarrow h^\dagger \left(Z[ip] + Z[ip]^\dagger \right) h \geq 0 \tag{9}$$

Using relation (7), the passivity property may equivalently be written

$$\mathrm{Re}\, p \geq 0 \Rightarrow h^\dagger \left(1 - S[ip]S[ip]^\dagger\right) h \geq 0 \tag{10}$$

By choosing particular vectors h with components 1 and 0 or 0 and 1, one deduces

$$\mathrm{Re}\, p \geq 0 \Rightarrow \left\{ \begin{array}{c} \left|r[ip]\right|^2 + \left|t[ip]\right|^2 \leq 1 \\ \left|\overline{r}[ip]\right|^2 + \left|t[ip]\right|^2 \leq 1 \end{array} \right\} \tag{11}$$

It follows immediately that all scattering coefficients of a passive mirror have a modulus smaller than unity for frequencies in the upper half of the complex plane. Coming back to the notations of equation (2), this theorem may be stated

$$\mathrm{Re}\, p \geq 0 \Rightarrow |r_j[ip]| \leq 1 \tag{12}$$

To evaluate the force (2), we use the equivalent expression

$$\begin{aligned} F &= \frac{\hbar}{c} \int_0^\infty \frac{\mathrm{d}\omega}{2\pi} \omega \left(-f[\omega] - f[\omega]^*\right) \\ f[\omega] &= \frac{r_1[\omega]r_2[\omega]e^{2i\omega\tau}}{1 - r_1[\omega]r_2[\omega]e^{2i\omega\tau}} = \frac{r_1[\omega]r_2[\omega]}{e^{-2i\omega\tau} - r_1[\omega]r_2[\omega]} \end{aligned} \tag{13}$$

The function $f[\omega]$ is the loop function corresponding to one roundtrip of the intracavity field. As a consequence of (12), the product of the reflection amplitudes of two passive mirrors has a modulus smaller than unity

$$\mathrm{Re}\, p \geq 0 \Rightarrow |r_1[ip]r_2[ip]| \leq 1 \tag{14}$$

It follows that the system "cavity (with motionless mirrors) plus vacuum" is stable and $f[\omega]$ is analytic in the upper half of the complex frequency plane. The force (13) may then be rewritten as an integral over imaginary frequency $\omega = ip$ with p real [13]

$$F = \frac{\hbar}{\pi c} \int_0^\infty \frac{p \; r_1[ip]r_2[ip]}{e^{2p\tau} - r_1[ip]r_2[ip]} \mathrm{d}p \tag{15}$$

We have used the fact that $f[ip]$ is real if p is real since $r_j[ip]$ is the Laplace transform of a real function [4]. In the limit of perfect reflection, expression (1) of the Casimir force is recovered

$$F_\mathrm{P} = \frac{\hbar}{\pi c} \int_0^\infty \frac{p}{e^{2p\tau} - 1} \mathrm{d}p \tag{16}$$

With the help of theorem (14), we deduce that the Casimir force (15) between two arbitrary passive mirrors has an absolute value smaller than the value (16) reached for perfect mirrors

$$|F| \leq F_\mathrm{P} \tag{17}$$

This is a first bound on the Casimir force which shows that the latter cannot be arbitrarily modified, because mirrors are passive systems.

4 The Casimir force between multilayer dielectric mirrors

We will now concentrate on the particular case of multilayer dielectric mirrors and derive first the properties obeyed by their reflection amplitudes from which we will then deduce further properties of the Casimir force.

We consider the multilayer dielectric mirror to be built of dielectric slabs. Each slab is symmetrical in the exchange of its two ports. Therefore the two reflection amplitudes contained in the S-matrix of a single slab are equal. The reflection and transmission amplitudes of a slab denoted by A are given by the following relations taken for example from [14] and translated from real to imaginary frequencies

$$\begin{aligned} r_{\mathrm{A}}[ip] &= -\frac{\rho_{\mathrm{A}}\left(1-e^{-2p\xi_{\mathrm{A}}}\right)}{1-\rho_{\mathrm{A}}^2 e^{-2p\xi_{\mathrm{A}}}} \\ t_{\mathrm{A}}[ip] &= \frac{\left(1-\rho_{\mathrm{A}}^2\right)e^{-p\xi_{\mathrm{A}}}}{1-\rho_{\mathrm{A}}^2 e^{-2p\xi_{\mathrm{A}}}} \\ \rho_{\mathrm{A}}[ip] &= \frac{\sqrt{\varepsilon_{\mathrm{A}}[ip]}-1}{\sqrt{\varepsilon_{\mathrm{A}}[ip]}+1} \\ \xi_{\mathrm{A}}[ip] &= \sqrt{\varepsilon_{\mathrm{A}}[ip]}\frac{l_{\mathrm{A}}}{c} \end{aligned} \tag{18}$$

These amplitudes have the same form as those of a Fabry-Perot cavity where ρ_{A} is the reflection amplitude for one interface and ξ_{A} an equivalent time of flight between the two interfaces (l_{A} is the thickness of the slab). The dielectric constant ε_{A}, evaluated for imaginary frequencies, approaches unity for $p \to \infty$ and is everywhere a real number larger than unity, as a consequence of Kramers-Kronig relations and passivity of the dielectric medium [15].

From relations (18) follows that the reflection amplitude of a dielectric slab is negative for $\mathrm{Re}\, p \geq 0$ while the transmission amplitude is positive. These latter properties are still fulfilled for multilayer mirrors as we will show in the following. As every multilayer mirror is built of dielectric slabs, it remains to proove that the upper properties are preserved when the slabs are piled up. To this aim we will represent each mirror by a transfer matrix which relates the fields on its left-hand and right-hand side (see Fig. 1)

$$\begin{bmatrix} a_{\mathrm{L}}^{\mathrm{out}} \\ a_{\mathrm{L}}^{\mathrm{in}} \end{bmatrix} = T \begin{bmatrix} a_{\mathrm{R}}^{\mathrm{in}} \\ a_{\mathrm{R}}^{\mathrm{out}} \end{bmatrix} \tag{19}$$

The explicit form of the T-matrix in terms of reflection and transmission amplitudes follows directly from expression (4) of the S-matrix

$$T = \frac{1}{t}\begin{bmatrix} t^2 - r\overline{r} & r \\ -\overline{r} & 1 \end{bmatrix} \tag{20}$$

A reciprocal mirror corresponds to a T-matrix having a determinant equal to unity.

Definition (19) is such that the T-matrix associated with a multilayer mirror is given by the product of the T-matrices of the elementary slabs which form the mirror (see Fig. 2). The T-matrix associated with the whole multilayer mirror

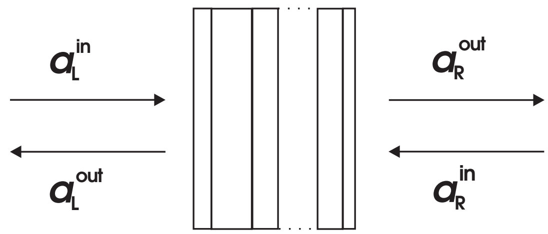

Figure 2: Multilayer mirror obtained by piling up elementary slabs.

may thus be obtained by iterating piling processes where an elementary slab is added to a given set of slabs. Denoting the extra slab by A and the set of slabs already piled up by B, the iteration step is then described by the relation

$$T_{\mathrm{AB}} = T_{\mathrm{A}} T_{\mathrm{B}} \tag{21}$$

As a consequence, the reciprocity property is preserved when slabs are piled up. This characteristic of electromagnetic networks [6] is only a particular feature of more general properties of reciprocal networks [12]. In contrast, the symmetry in the exchange of the two ports which is obeyed by a single slab is no longer satisfied by multilayer mirrors. Reflection and transmission amplitudes are obtained by developing equation (21) with the help of (20)

$$\begin{aligned} t_{\mathrm{AB}} &= \frac{t_{\mathrm{A}} t_{\mathrm{B}}}{1 - \overline{r}_{\mathrm{A}} r_{\mathrm{B}}} \\ r_{\mathrm{AB}} &= r_{\mathrm{A}} + \frac{r_{\mathrm{B}} t_{\mathrm{A}}^2}{1 - \overline{r}_{\mathrm{A}} r_{\mathrm{B}}} \\ \overline{r}_{\mathrm{AB}} &= \overline{r}_{\mathrm{B}} + \frac{\overline{r}_{\mathrm{A}} t_{\mathrm{B}}^2}{1 - \overline{r}_{\mathrm{A}} r_{\mathrm{B}}} \end{aligned} \tag{22}$$

It immediately follows that the reflection amplitudes evaluated at imaginary frequencies for a multilayer dielectric mirror are negative while the transmission amplitudes are positive.

Coming back to the notation of equation (2) we deduce in particular that the reflection amplitude r_j associated with each mirror is negative

$$\mathrm{Re}\, p \geq 0 \Rightarrow r_j[ip] \leq 0 \tag{23}$$

The product of the reflection amplitudes of the two mirrors is thus positive

$$\mathrm{Re}\, p \geq 0 \Rightarrow 0 \leq r_1[ip] r_2[ip] \leq 1 \tag{24}$$

It follows that the Casimir force (15) between two arbitrary multilayer dielectric mirrors is attractive for any cavity length and decreases as a function of the length

$$
\begin{aligned}
&0 \leq F \leq F_{\mathrm{P}} \\
&\frac{\mathrm{d}F}{\mathrm{d}\tau} \leq 0
\end{aligned}
\tag{25}
$$

These properties constitute further bounds for the Casimir force valid for any multilayer dielectric mirror. One may remind at this point that a repulsive Casimir force may be obtained by considering a cavity built with a dielectric and a magnetic plates [16]. The product r of the two reflection amplitudes is indeed negative in this case, so that the force (15) is repulsive.

5 The Casimir force between narrow-band mirrors

One may give a simple and accurate estimation of the Casimir force in the particular case of narrow-band multilayer dielectric mirrors, i.e. of mirrors having an optical depth much smaller than the distance between them.

This possibility is based on the existence of a suitable approximation for the typical variation of the reflection amplitudes as functions of frequency. For a single slab, the reflection amplitude given in equation (18) vanishes at zero frequency $p = 0$ and remains small for frequencies such that $p\xi_{\mathrm{A}} < 1$. This property has a simple interpretation. At low frequencies the wavelength of the field is indeed larger than the optical depth of the slab, so that the slab appears to the field as a smooth modulation of the refractive index rather than as an abrupt interface. The reflection amplitude $r_{\mathrm{A}}[ip]$ thus vanishes at zero frequency while the transmission amplitude is unity at zero frequency (cf. eqs (18)). The behaviour of the reflection coefficient at small frequencies is therefore mainly determined by its derivative at zero frequency $r'_{\mathrm{A}}[0]$

$$
\begin{aligned}
r_{\mathrm{A}}[ip] &\simeq -p\theta_{\mathrm{A}} \\
\theta_{\mathrm{A}} &= -ir'_{\mathrm{A}}[0]
\end{aligned}
\tag{26}
$$

We have introduced here a coefficient θ_{A} which can be easily evaluated from (18) for a single slab

$$
\theta_{\mathrm{A}} = \frac{\varepsilon_{\mathrm{A}} - 1}{2} \frac{l_{\mathrm{A}}}{c}
\tag{27}
$$

This coefficient is always positive and is similar to an optical depth of the dielectric slab, however not measured as a length but as a time.

We can now use transformation rules (22) to derive the corresponding properties of a multilayer dielectric mirror. We first deduce that its reflection and

transmission amplitudes are still respectively zero and unity. Furthermore it follows from (22) that the derivatives of the reflection amplitudes at $p = 0$ are simply added when an extra slab is piled up onto a mirror

$$\theta_{\mathrm{AB}} = \theta_{\mathrm{A}} + \theta_{\mathrm{B}} \tag{28}$$

The reflection amplitude of a multilayer dielectric mirror is therefore still given by the approximated expression (26), with a coefficient θ now written as a sum over the multiple layers A

$$\theta = \sum_{\mathrm{A}} \frac{\varepsilon_{\mathrm{A}} - 1}{2} \frac{l_{\mathrm{A}}}{c} \tag{29}$$

Coefficient θ is also closely related to the mirrors bandwidth. By using the usual dispersion relation for the reflection amplitude, we can indeed write

$$\theta = \frac{2}{\pi} \int_0^\infty \mathrm{d}\omega \frac{(-\operatorname{Re} r[\omega])}{\omega^2} \tag{30}$$

In the limit of narrow bandwidth in particular, it is directly proportional to the bandwidth for mirrors having a given central reflection frequency.

We may now deduce a simple approximation of the Casimir force (15) between two narrow-band mirrors ($\theta_1 \ll \tau$ and $\theta_2 \ll \tau$). In the integral (15), the dominant contribution indeed comes from low frequencies, due to the exponential factor $e^{2p\tau}$ appearing in the denominator. This integral may thus be approximated in terms of the coefficients θ_j

$$\begin{aligned} F &\simeq \frac{\hbar}{\pi c} \int_0^\infty p \; r[ip] \; e^{-2p\tau} \mathrm{d}p \\ &\simeq \frac{\hbar \theta_1 \theta_2}{\pi c} \int_0^\infty p^3 \; e^{-2p\tau} \mathrm{d}p \\ &\simeq \frac{3\hbar \theta_1 \theta_2}{8\pi c \tau^4} \end{aligned} \tag{31}$$

The Casimir force for narrow-band mirrors (31) can now be compared to the force evaluated in the limit of perfectly reflecting mirrors (1)

$$F = F_{\mathrm{P}} \frac{9\theta_1 \theta_2}{\pi^2 \tau^2} \tag{32}$$

Clearly the Casimir force between narrow-band mirrors is always much smaller than between perfect mirrors. In particular it decreases proportionally to the product of the reflection bandwidths of the two mirrors.

6 Discussion

As discussed in the Introduction, the Casimir force results from a fine balance between repulsive and attractive contributions associated respectively with resonant and antiresonant modes of vacuum fluctuations. This might lead to expect

that specific frequency dependences designed to affect this balance could allow to change quantitative properties of the Casimir force. We have shown in the present letter that such expectations cannot be met due to fundamental properties obeyed by real mirrors. For any mirror obeying passivity properties, the Casimir force never exceeds the limiting value obtained for perfect mirrors. For multilayer dielectric mirrors, even tighter bounds are obtained for the Casimir force. It remains attractive for any cavity length while its value is a decreasing function of the cavity length.

This means that the arguments presented in the Introduction fail despite of their apparent pertinence. A qualitative interpretation of this failure may be drawn by recalling that resonances are in fact determined by phase conditions. Besides the phase-shift due to free flight between the two mirrors, the field also undergoes phase-shifts during reflection on the mirrors. Equation (2) shows that these phase-shifts induce a cavity resonance shift which was disregarded in the simple discussion presented in the Introduction. For narrow-band mirrors in particular, the dephasing of the field differs by a phase π above and below resonance, due to analyticity properties of the reflection amplitudes. A phase-shift of π is exactly the value required for shifting the cavity detuning from one mode to the next one. This phase-shift thus forbids to separate repulsive and attractive contributions to the Casimir force and, therefore, to meet the expectation of a more sensitive dependence of the Casimir force versus distance.

These results should be considered in order to assess the feasibility of novel experimental demonstrations of the Casimir effect with frequency dependent mirrors. From the conceptual point of view, they also raise interesting questions by revealing intimate connections between causality and passivity properties on one side, and the evaluation of the storage of vacuum energy by a cavity on the other side.

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References

[1] H.B.G. Casimir, *Proc. K. Ned. Akad. Wet.* **51** 793 (1948); G. Plunien, B. Müller and W. Greiner, *Phys. Rep.* **134** 87 (1986).

[2] L.S. Brown and G.J. Maclay, *Phys. Rev.* **184** 1272 (1969).

[3] S. Haroche in "*New Trends in Atomic Physics*", G. Grynberg and R. Stora eds. (North Holland, 1984) p. 193.

[4] M.T. Jaekel and S. Reynaud, *J. de Physique* **I-1** 1395 (1991).

[5] E. Iacopini, *Phys. Rev.* **A48** 129 (1993).

[6] J. Meixner, *J. of Math. Phys.* **4** 154 (1963); J. Meixner in *Statistical mechanics of equilibrium and non-equilibrium* ed. J. Meixner (North Holland, 1965) p. 52; J. Meixner, *Acta Phys. Polonica* **27** 113 (1965).

[7] "*Quantum Noise Reduction in Optical Systems*", eds E. Giacobino and C. Fabre, *Appl. Phys.* **B55** 189 (1992).

[8] D. Slepian, *Bell Sys. Tech. J.* **43** 3009 (1964).

[9] L. Landau and E.M. Lifshitz, "*Electrodynamics of continuous media*" (Pergamon Press) §90.

[10] D. Kupiszewska, *Phys. Rev.* **A46** 2286 (1992); the force obtained in this paper is twice larger than formula (2) because the contributions of the two field polarisations have been added.

[11] S.M. Barnett, C.R. Gilson, B. Huttner and N. Imoto, *Phys. Rev. Lett.* **77** 1739 (1996).

[12] L. Onsager, *Phys. Rev.* **37** 405 (1931), **38** 2265 (1931); H.B.G. Casimir, *Rev. of Mod. Phys.* **17** 343 (1945).

[13] E.M. Lifshitz, *Sov. Phys. JETP* **2** 73 (1956) [*Zh. Exp. Teor. Fis.* **29** 94 (1955)].

[14] A.E. Siegman, "*Lasers*" chap. 11.1 (University Science Books, 1986)

[15] L. Landau and E.M. Lifshitz, "*Electrodynamics of continuous media*" (Pergamon Press) §62.

[16] T.H. Boyer, *Phys. Rev.* **A9** 2078 (1974); D. Kupiszewska, *J. of Mod. Opt.* **40** 517 (1993).